

Kinematics and Automation of Six-Strut Systems using Linear Actuators

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Abstract: Six-Strut Systems is a parallel manipulator designed at Lawrence Berkeley National Laboratory. It enables precise translations in X, Y, and Z, and rotations in yaw, pitch, and roll. Adjusting one strut causes a coupled movement in the other struts. Currently, the struts are manual; to automate them, linear actuators and kinematics for six-strut systems are needed. Using a rotation matrix and defining two frames, one for the object and one for the fixed frame, equations are made to calculate the strut lengths. With these lengths, linear actuators can be programmed to automate the strut adjustments.

Introduction:

Parallel manipulators are crucial in many industries and provide multi-degree movement with high precision for various applications. However, the kinematics and automation of these parallel manipulators are not thoroughly explained. Parallel manipulators are closed-loop structures where a platform connects to fixed points using serial limbs to provide multi-degree freedom to the platform.¹ Different manipulators will require different numbers and configurations of limbs resulting in varying degrees of freedom (DOF). Our six-strut system is a parallel manipulator and allows for six DOF with six struts. The six-strut system, developed at Lawrence Berkeley National Laboratory, is used to hold magnets for the Advanced Light Source (ALS). A single strut consists of a fixed end and a movable end, with one end of the rod attached to a fixed surface and the other end attached to a plate or object that is supported by the struts.

Our strut can be defined as a rigid member with spherical joints on each end.²

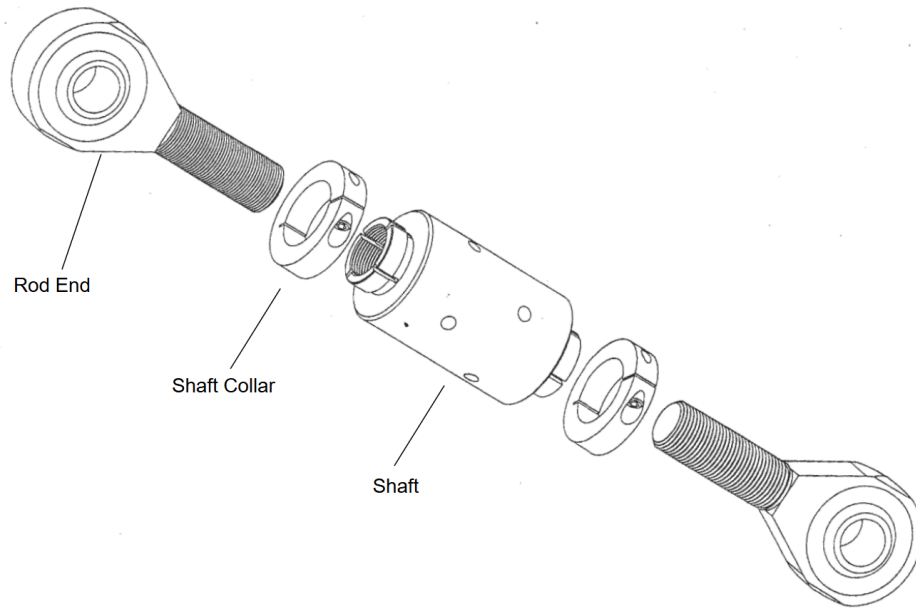


Fig 1. Exploded Strut View²

The spherical joints allow for higher DOFs but require exactly six struts for the platform to be stable.³ Figure 2 represents a potential design for a six strut system. If there were less than 6 struts the system will instantly be unstable and tip over since struts are configured to give support and control in each DOF. Changing the length of an individual strut allows for the platform connected to move in six DOF. Our six-strut system is manually adjusted, which allows for cheaper costs. However, the lack of automation is time-consuming on six-strut systems that need frequent adjustments.

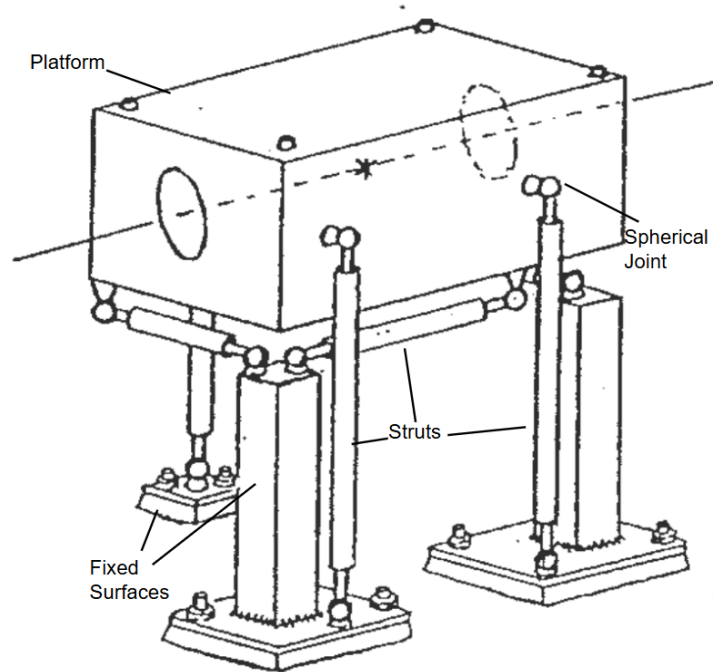


Fig 2. Example Six Strut System³

A single strut consists of two threaded rod end bearings with different thread directions.² The two rod ends are screwed onto either side of a shaft with corresponding threads for each rod. When twisting the shaft, while preventing the two rod ends from twisting, the strut will change in length. This functions as a turnbuckle, which allows the rods to lengthen and shorten when twisted. Some struts incorporate a thread differential; which enables precise movements.⁴ Thread differentials have threads with two different types of pitches. The difference in these pitches causes a relative movement between the threads, which provides a mechanical advantage equal to the difference in the pitches. For large designs, a thread differential is not necessary, as the pitch can be larger when the rods are longer. Figure 1 shows an exploded view of a strut with the threaded rod ends. Each rod end is connected with a truncated ball bearing with a through-hole for attachment. The distance between the centers of the truncated bearings determines the length

of the rod. Connected to each truncated bearing is a strut boss that connects the rod to the platform or object and fixed locations with a bolt.

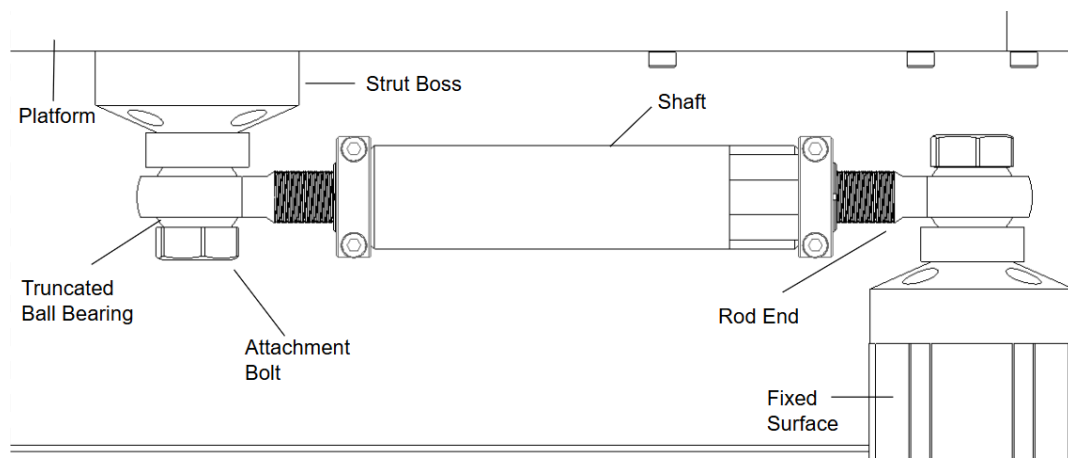


Fig 3. Fully Assembled and Attached Strut

When adjusting a single strut, other struts also move causing unwanted coupling effects of translations and rotations. Correcting for these unwanted movements requires an inefficient trial-and-error approach. To automate the process requires knowledge of the desired position and the necessary strut lengths. By defining the geometry and using the known dimensions of the six-strut system, a calculator was made that outputs the necessary strut lengths for the desired rotation and translation of the platform or object. This calculator will work for any form factor of a six-strut system as long as the correct values are inputted. Once the parameters are set, the desired position and rotation is the input with output as the necessary strut lengths for each strut.

With knowledge of the necessary lengths and positions for each strut, the current manual struts can be replaced with automated struts. Furthermore, Automated struts are available in various size and power ratings enabling the six-strut systems to be used in large scale and heavy load applications. Using a controller, the required strut lengths for the desired position can be inputted. Designing a custom automated six-strut will not be cheaper than the manual design.

However, a custom design will still be cheaper than commercially available designs and enable scalability for larger manipulators.

Materials:

Equations are derived to determine the positions of the ball joints of the free ends after changes in orientation and position of the object. The equations are then implemented in Excel to facilitate easy input of parameters and calculation of final strut lengths. The equations themselves output the end positions of the free ball joints relative to the object. The coordinates of the fixed ball joints are measured and input into Excel. Using the positions of the two balls, the length of the strut can be calculated. In total, the coordinates of 12 ball joints and the position of the object must be collected for the calculator to work accurately. To accurately collect the coordinates, CREO Parametric will be utilized.

CREO Parametric will contain a CAD assembly of the six-strut system, and its measuring tool will be used to record all positions with a defined axis. CREO Parametric will also be used to validate the Excel calculator and derived equations. Additionally, CREO Parametric will be used to design a model to automate the strut system, and ANSYS will be used to determine the structural stability of the design.

Methodology:

The length of the strut is defined as the distance between the two truncated bearings. The fixed ball joint always maintains the same position. In contrast, the second ball joint moves with respect to the object being rotated and translated. Two frames are created: one for the moving object and one for world space, which is a fixed frame. The position of the free ball joint does not change with respect to the moving frame, as the object moves and translates with it.

However, the position of the free ball joint does change with respect to the fixed frame. Using a rotation matrix, the position in the fixed frame can be determined from the moving frame.

The rotation matrix and position of the ball can be used to derive equations for the X, Y, and Z coordinates of the free ball joint in the fixed frame. In the fixed frame, the positions of the two ball joints are used to create a vector. The magnitude of this vector represents the length of the strut. By repeating the calculations for all six struts, we can determine the necessary strut lengths when applying a desired rotation and translation to an object. The required initial positions include the object's position with respect to the fixed frame, the free ball's position with respect to the object, and the fixed balls' positions with respect to the fixed frame. Additionally, Excel can take the user's desired rotation and translation as input and output the strut lengths using the rotation matrix.

Using CREO and overlaying two assemblies of the six-strut system can verify the Excel calculations, which in turn verify the equations and rotation matrix. By creating two identical assemblies with the same orientation and overlaying them on top of each other, one assembly can be rotated and translated while the other remains fixed in space. The length of the rod can be determined by measuring the distance from the free bearing of the displaced assembly to the fixed bearing of the stationary assembly. If inputting the same rotation and translation yields the same strut lengths as measured in CREO, it confirms that the equations and Excel calculations are correct.

Results:

To derive the rotation matrix, two frames are defined: a fixed frame and an object frame that rotates. In a six-strut system, the object, plate, and free ball joints move together. Vectors

from the object frame to the free ball joints remain constant, while vectors from the fixed frame to the free ball joints change with rotation. A rotation matrix is used to transform vectors between frames. Rotation is described by alpha (α), beta (β), and gamma (γ) around the X, Y, and Z axes, respectively. The rotation matrix is formed by multiplying rotation matrices for single-axis rotations.

$$R_M = \begin{bmatrix} X & Y & Z \\ \cos(\gamma)\cos(\beta) & \sin(\alpha)\sin(\beta)\cos\gamma + \sin(\gamma)\cos(\alpha) & -\cos(\alpha)\cos(\gamma)\sin(\beta) + \sin(\gamma)\sin(\alpha) \\ -\cos(\beta)\sin(\gamma) & -\sin(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma) & \cos(\alpha)\sin(\beta)\sin(\gamma) + \sin(\alpha)\cos(\gamma) \\ \sin(\beta) & -\sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) \end{bmatrix} \quad (1)$$

Using the rotation matrix, three equations can be formed to convert the ball joint position in the free frame to the position in the fixed frame after rotation. The rotation matrix represents the fixed coordinate position (X_F, Y_F, Z_F) in its columns. By multiplying the coordinates of the free ball joints (X_O, Y_O, Z_O) with respect to the free frame with the columns, the coordinates in the fixed frame are obtained.

$$X_F = [\cos(\gamma)\cos(\beta)] * X_O - [\cos(\beta)\sin(\gamma)] * Y_O + [\sin(\beta)] * Z_O \quad (2)$$

$$\begin{aligned} Y_F &= [\sin(\alpha)\sin(\beta)\cos(\gamma) + \sin(\gamma)\cos(\alpha)] * X_O \\ &+ [-\sin(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\cos(\gamma)] * Y_O \\ &- [\sin(\alpha)\cos(\beta)] * Z_O \end{aligned} \quad (3)$$

$$\begin{aligned} Z_F &= [-\cos(\alpha)\cos(\gamma)\sin(\beta) + \sin(\gamma)\sin(\alpha)] * X_O \\ &+ [\cos(\alpha)\sin(\beta)\sin(\gamma) + \sin(\alpha)\cos(\gamma)] * Y_O \\ &+ [\cos(\alpha)\cos(\beta)] * X_O \end{aligned} \quad (4)$$

Using these equations, Excel streamlines the inputs and calculations. Excel takes the initial positions of the free ball joints relative to the object frame and the fixed ball joints relative to the fixed frame. Additionally, the position of the object frame relative to the fixed frame is required, unless the fixed frame is arbitrarily placed at the origin of the object frame, making this step unnecessary. With equations 2, 3, and 4, the coordinates of the free ball joint in the fixed frame are determined. Knowing the positions of both ball joints, the distance between them is calculated to obtain the strut length.

Linear actuators will serve as automatic alternatives to the struts. Given the diverse range of six-strut systems and applications, linear actuators with varying specifications are required. The struts can be categorized into four groups: two with a load capacity of approximately 100 kg and stroke lengths of 30 mm and 50 mm, respectively, and two with a load capacity of around 500 kg and stroke lengths of 30 mm and 50 mm. Multiple linear actuators were selected, offering various costs and specifications. For example, the Electrak LL24 series actuators from Thomson Linear support loads of up to 2000 N and 4000 N with stroke lengths of up to 100 mm.⁵

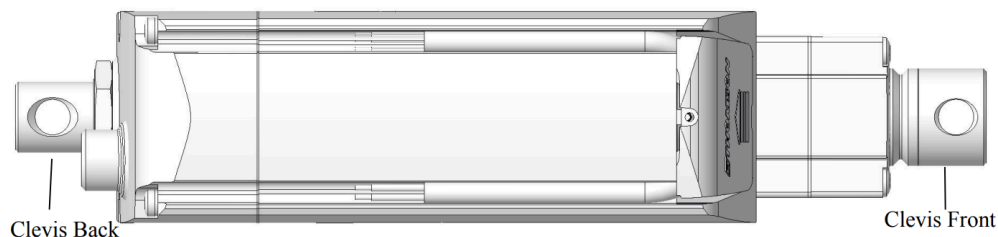


Fig 4. Thomas Linear Electrak LL24 Actuator⁵

Most linear actuators have a clevis fixture at both ends, but Six-Strut systems require ball joints for six DOF. A custom attachment piece is designed to fit over the clevis end, allowing a ball bearing rod end to be threaded in. Separate pieces are created for the front and back fixtures, secured by a bolt through the clevis.

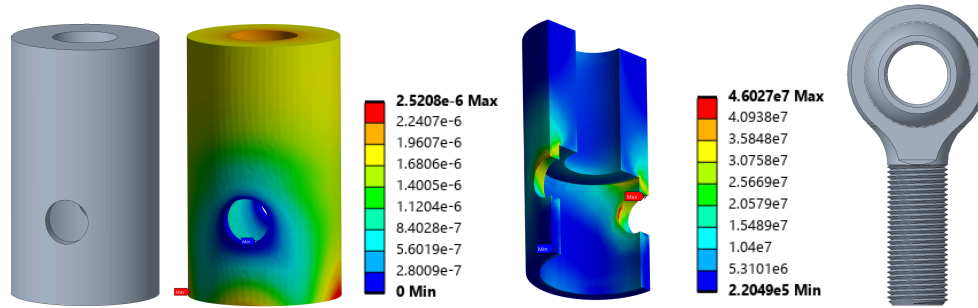


Fig 5. Elactrak LL24 Attachment CAD, Deformation, and Stress with Rod End⁶

ANSYS Mechanical was used to run a static structural analysis on all the attachment pieces connecting the threaded bearing end to the clevis. Figure 6 shows the front attachment for Elacktrak LL24 where a load of 5000N was applied on the inner tube of the attachment representing the force from the threads of the rod end. All attachments are made with stainless steel.

Table 1. Max Deformation for Clevis-Rod Respective to Actuator Spec Ratings

	Elacktrak LL24	LA23	McMaster (4267N23)	McMaster (4492N18)
Deformation Front (m)	2.5e-6	1.7e-5	1.16e-6	2.5e-6
Deformation Back	5.7e-7	1.1e-6	6.1e-8	1.6e-6

Notes: The attachments are fixed at the side holes to represent a bolt securing the attachment to the clevis. The force is applied downward on the inner walls of the top center hole. The force applied is 500N above the rated force of the actuators.

Conclusion

The inefficiencies of a manual six-strut system, caused by the coupling effect, can be addressed with an automated system. A custom automated six-strut system offers cost and time savings while eliminating manual inefficiencies. Linear actuators can serve as automatic struts, but require modification to match manual strut functionality. Various linear actuator models were selected to design struts with different capabilities. While linear actuators do not directly mitigate the coupling motion, automation enables rapid compensation for the effect, making the effect obsolete when compared to manual adjustments. A custom attachment was designed to connect a ball bearing rod end to the linear actuator's clevis end. The attachments were stress-tested in ANSYS to ensure they can withstand applied forces. Kinematic equations, derived from a rotation matrix between a moving and fixed frame, enable calculation of necessary actuator lengths for a desired position and rotation. These equations can be programmed into a controller to automate the six-strut system, controlling the linear actuators' motors.

Acknowledgements

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